

An Uncertain Resource Constrained Scheduling Model Based on Uncertainty Theory

Qian Zhang¹, Xiaosheng Wang^{2,*}, Wei Li², Hao Hu¹

¹School of Management Engineering and Business, Hebei University of Engineering, Handan, China

²School of Mathematics & Physics, Hebei University of Engineering, Handan, China

Email address:

876332890@qq.com (Qian Zhang), xswang@hebeu.edu.cn (Xiaosheng Wang), liwei066000@126.com (Wei Li),

462013808@qq.com (Hao Hu)

*Corresponding author

To cite this article:

Qian Zhang, Xiaosheng Wang, Wei Li, Hao Hu. An Uncertain Resource Constrained Scheduling Model Based on Uncertainty Theory. *Mathematics and Computer Science*. Vol. 4, No. 6, 2019, pp. 138-141. doi: 10.11648/j.mcs.20190406.15

Received: November 20, 2019; **Accepted:** December 16, 2019; **Published:** December 27, 2019

Abstract: Resource constrained project scheduling problem is to make a schedule for minimizing of the completion time or total cost subject to precedence rules and resource constraints. Traditional resource constrained project scheduling problem research takes into account achieve management goal in certain environment. However, there are many uncertainties in practical projects due to the uncertain factors, which leads to the change of resource availability. In this paper, for better described the uncertain resource constrained project scheduling problem, we firstly consider the uncertain resource availability project scheduling problem based on uncertainty theory. To meet the manger goals, it is assumed that the increased quantities of resource are uncertain variables and the finish time of each activity is a decision variable. Then, an uncertain resource constrained model is built. The goals of the model are to minimize the completion time and the total cost which composed by the activity cost and the additional resource cost. One of the constraints is the finish-start precedence relationship among the project activities. The other constraint is the resource constraint in which the demand of resource shall not exceed the total supply of resource for each resource type at any time. Furthermore, the equivalent form of the above model is given and its equivalence is proved. Finally, a genetic algorithm is applied to search for quasi-optimal scheduling, and a project example is given to illustrate the effectiveness of the model.

Keywords: Project Schedule Problem, Uncertainty Theory, Uncertain Resource

1. Introduction

The resource constrained project scheduling problem (RCPSP) takes into account the balance of makespan and total cost through resource allocation and reasonable activity schedules while the precedence constrains between the activities and resource constrains are all satisfied. In recent years, many scholars discussed different types of resource constrained project scheduling problems, such as multi-mode RCPSR [1-3], multi-project RCPSP [4-5], robust RCPSP [6-7], and so on.

However, the majority of the above studies focus on RCPSP in deterministic environment, and suppose that the resource availability is a real number. In real-word projects, there may be some uncertainty phenomena, for example, overdue materials, the decrease in the number of workers at Grain in

Ear season, etc, which result in the resource availability may be changed. In this case, many scholars begin to consider the uncertain resource constrained project scheduling problem. Ying [8] proposed a schedule model of flexible work-hour constraint, in which the human resource was dealt with a new constraint to the classical RCPSP and the increased quantities of human resource were real-value variables. However, the cost that effected by the increased quantities of human resource was not taken into account. Xie [9] supposed that the increased quantities of resource were real-value variables, and built a multi-mode resource constrained project scheduling model for minimizing both the project makespan and cost. Lambrechts [10] established a stochastic project scheduling model in which the resource availability was a random variable in order to increase robustness. Its goal was minimizing the expected weighted sum of the absolute deviations between the planned and the

actually realized activity starting times. Its constrains were the resource and priority rules. Chen [11] developed a project scheduling problem model under fuzzy resource constrained, of which the fuzzy duration time and fuzzy resource availability were represented by triangular fuzzy number.

When the indeterminacy does not behave neither randomness nor fuzziness, a new tool is required to deal with it. Uncertainty theory based on uncertain measure founded by Liu [12], and its a branch of axiomatic mathematics for modeling human uncertainty. Liu [13] firstly established an uncertain project scheduling model, aiming to minimize the total cost under the constraint that the completion time does not exceed the deadline. Ji and Yao [14] recently considered the uncertainty of the duration times and the resources allocation times by assuming them are uncertain variables. Ma [15] considered resource constrained project scheduling problem with uncertain durations, and an uncertain excepted value model was built with the objective was to minimize the completion time.

Up to now, we have not yet found uncertain resource availability constrained RCPSp in uncertain environment, which is not either randomness or fuzziness. In this paper, we consider uncertain resource availability project scheduling problem based on uncertainty theory, in which the increased quantities of resource are uncertain variables, and the finish time of each activity is a decision variable. Then, we build a multi-objective model which is under resource and precedence rule constrained to minimize the resource cost and the project completion time. For solving the above model, the equivalent form of the model is provided and the proof is given. Furthermore, a project example is proposed and the optimal scheduling scheme of the project is obtained by genetic algorithm.

The construction of this paper is organized as follows. In Section 2, an uncertain resource constrained project scheduling model will be built and transformed into a crisp form. Then, we will illustrate the validity of the above model via a numerical example in Section 3. Finally, some concluding statements will be covered in Section 4.

2. Formulation and Model

2.1. Problem Description

With the project implement, the shortage of resources is one of the most critical influences on project process. In order to solve the problem of time-cost trade-off in project scheduling, we consider a project which is described as an activity-on-the-node network $G(N,A)$, where $N = \{1, 2, \dots, n\}$ is the set of activities and A is the set of pairs of activities with precedence relations. Node 1 and node n represent the start and the end of the project, respectively. A multi-objective programming model is built under the resource restricts and precedence rules of activities constrains in order to balance the completion time and the total cost of the project.

2.2. The Basic Assumptions

- (1) This paper only considers renewable resources.
- (2) Suppose that the increased quantities of resource are independent uncertain variables with regular uncertainty distributions.
- (3) Assume that the finish time of each activity is a decision variable.
- (4) No interruption is allowed for each activity in progress.

2.3. Symbol and Variable Interpretation

- f_i : Finish time of activity i , $1 \leq i \leq n$;
- d_i : Duration time of activity i , $1 \leq i \leq n$;
- R_k : Availability of resource k , $1 \leq k \leq K$;
- r_{ik} : Requirement of resource k by activity i ;
- R'_k : The increased quantities of resource k , $1 \leq k \leq K$;
- c_i : The cost of activity i ;
- c'_{R_k} : The cost per time unit of additional resource R'_k ;
- A_t : The set of underway activities at time t .

2.4. Uncertain Resource Constrained Time-Cost Trade-off Model

For some time-intensive and heavy-duty projects, managers tend to be completed as quickly as possible and cost to be minimized. Managers usually increase resource availability to improve construction efficiency and shorten construction time, but it often makes the projects with high costs. Therefore, managers always required to make trade-off between the total cost and the completion time. The uncertain resource constrained time-cost trade-off problem can be described as following optimization model:

$$\left\{ \begin{array}{l} \min f_n \textcircled{1} \\ \min E\{c\} \textcircled{2} \\ \text{s. t.} \\ f_j - d_j \geq f_i, (i, j) \in A \textcircled{3} \\ \mathcal{M}\{\sum_{i \in A_t} r_{ik} \leq R_k + R'_k\} \geq \alpha_0 \textcircled{4} \\ C = \sum_{i=1}^n c_i + \sum_{k=1}^K c'_{R_k} \cdot f_n \cdot R'_k \textcircled{5} \\ f_i \geq 0, i \in N \textcircled{6} \end{array} \right. \quad (1)$$

In the above model, objective $\textcircled{1}$ is to minimize the project total completion time; Objective $\textcircled{2}$ is to minimize the expected project total cost which consists of the activity cost and the additional resource cost. Constraint $\textcircled{3}$ declares that finish-start precedence relation among project activities. As to a pair of activity i and j , activity j start after its predecessor activity i is finished. Constraint $\textcircled{4}$ reflects that for any time t and each resource type k , the demand for resources shall not exceed the total supply of resources with at least given confidence level α_0 . Constraint $\textcircled{5}$ illustrates the total cost of project has two parts. The one is original resources cost determined by resource employ. The other is additional resource costs, in which means the product of the cost per time unit of additional resource, project completion time and the increased quantities of resource k . Constraint $\textcircled{6}$ shows the range of decision variables.

In order to transform the model into deterministic form, we introduce the following several theorems.

Theorem 1. [16] Let ξ be an uncertain variable with regular uncertainty distribution Φ . If the expected value exists, then

$$E[\xi] = \int_0^1 \Phi^{-1}(\alpha) d\alpha. \quad (2)$$

For instance, let ξ be a linear uncertain variable, it has inverse uncertainty distribution $\Phi^{-1} = (1 - \alpha)a + ab$. Then expected value of ξ is

$$E[\xi] = \int_0^1 (1 - \alpha)a + ab d\alpha. \quad (3)$$

Theorem 2. [16] Let $\xi_1, \xi_2, \dots, \xi_n$ be independent uncertain variables with regular uncertainty distributions $\Phi_1, \Phi_2, \dots, \Phi_n$, respectively. If the function $g(x_1, x_2, \dots, x_n)$ is strictly increase with respect to x_1, x_2, \dots, x_m and strictly decreasing with respect to $x_{m+1}, x_{m+2}, \dots, x_n$, then

$$\mathcal{M}\{g(\xi_1, \xi_2, \dots, \xi_n) \leq 0\} \geq \alpha, \quad (4)$$

holds if and only if

$$g(\Phi_1^{-1}(\alpha), \Phi_2^{-1}(\alpha), \dots, \Phi_m^{-1}(\alpha), \Phi_{m+1}^{-1}(1 - \alpha), \dots, \Phi_n^{-1}(1 - \alpha)) \leq 0. \quad (5)$$

To solve the model (1), according to the operational law of uncertain variables, we transformed it into its equivalent form, as shown below.

Theorem 3. Model (1) is equivalent to the following model.

$$\left\{ \begin{array}{l} \min f_n \\ \min \int_0^1 Y^{-1}(\alpha) d\alpha \\ s. t. \\ f_j - d_j \geq f_i, (i, j) \in A \\ \Phi^{-1}(1 - \alpha_o) \geq \sum_{i \in A_t} r_{ik} - R_k \\ Y^{-1}(\alpha) = \sum_{i=1}^n c_i + \sum_{k=1}^K c'_{R_k} \cdot f_n \cdot \Phi^{-1}(\alpha) \\ f_i \geq 0, i \in N \end{array} \right. \quad (6)$$

Proof: Because R'_k is an uncertain variable with regular uncertainty distribution Φ_k , and

$$C = \sum_{i=1}^n c_i + \sum_{k=1}^K c'_{R_k} \cdot f_n \cdot R'_k, \quad (7)$$

by the definition of uncertain variable [16], we know that C is an uncertain variable, and the inverse uncertainty distribution of C is

$$Y^{-1}(\alpha) = \sum_{i=1}^n c_i + \sum_{k=1}^K c'_{R_k} \cdot f_n \cdot \Phi_k^{-1}(\alpha). \quad (8)$$

By Theorem 1, we know that the expected value of C is

$$E[C] = \int_0^1 \sum_{i=1}^n c_i + \sum_{k=1}^K c'_{R_k} \cdot f_n \cdot \Phi_k^{-1}(\alpha) d\alpha. \quad (9)$$

Since

$$\mathcal{M}\{\sum_{i \in A_t} r_{ik} \leq R_k + R'_k\} \geq \alpha_o, \quad (10)$$

then,

$$\mathcal{M}\{\sum_{i \in A_t} r_{ik} - R_k - R'_k \leq 0\} \geq \alpha_o. \quad (11)$$

By Theorem 2, we have

$$\sum_{i \in A_t} r_{ik} - R_k - \Phi_k^{-1}(1 - \alpha) \leq 0, \quad (12)$$

i.e.

$$\Phi_k^{-1}(1 - \alpha) \geq \sum_{i \in A_t} r_{ik} - R_k. \quad (13)$$

Therefore, the model (1) is equivalent to the model (2).

3. Case Study

In this section, we give a project example which is the decoration of engineering in the bonded areas data center to illustrate the reasonability of the model. The type of resources chosen in this case is human resource. For every activity i , the increased quantity of resource k is assumed to be a linear uncertain variable $L(3, 7)$. The duration time, cost and resource requirement of activities are presented in Table 1. The manager tends to the demand of resource shall not exceed the total resource supply with at least given confidence level 0.9 for any time t and each resource type k . The cost per time unit of additional resource c'_{R_k} is 280 yuan. The goals of project are both completion time minimization and expected total cost minimization. The constrains are recourse constraint and precedence constraint. The decision variable is finish time of each activity.

Table 1. The information of the activities.

Activity code	Duration	Worker	Cost	Preceding activity
1	0	0	0	0
2	1	5	1200	0
3	5	24	28800	2
4	27	16	103680	3
5	30	24	172800	3
6	22	24	126720	4
7	19	9	41040	4
8	24	6	34560	3
9	12	4	11520	8
10	22	20	105600	7
11	7	8	13440	6
12	20	8	38400	7
13	24	17	97920	5
14	12	6	17280	4
15	16	4	15360	7
16	11	6	15840	6
17	10	8	19200	14
18	8	6	11520	2
19	24	11	63360	18
20	13	4	12480	6
21	9	4	8640	12/15
22	3	4	2880	21
23	4	4	3840	7
24	0	0	0	22

With the above demand, we can present the following model:

$$\left\{ \begin{array}{l} \min f_{24} \\ \min E\{c\} \\ s. t. \\ f_j - d_j \geq f_i, (i, j) \in A \\ \mathcal{M}\{\sum_{i \in A_t} r_{ik} \leq 30 + R'_k\} \geq 0.9 \\ C = \sum_{i=1}^{24} c_i + 280 \cdot f_{24} \cdot R'_k \\ f_i \geq 0, i \in N \end{array} \right. \quad (14)$$

This model is equivalent to

$$\left\{ \begin{array}{l} \min f_{24} \\ \min \int_0^1 Y^{-1}(\alpha) d\alpha \\ s. t. \\ f_j - d_j \geq f_i, (i, j) \in A \\ \Phi^{-1}(1 - 0.9) \geq \sum_{i \in A_t} r_{ik} - 30 \\ Y^{-1}(\alpha) = \sum_{i=1}^{24} c_i + 280 \cdot f_{24} \cdot \Phi^{-1}(\alpha) \\ f_i \geq 0, i \in N \end{array} \right. \quad (15)$$

After a run 6000 generations with *op-size* = 300, $P_m = 0.1$, $P_c = 0.9$, the quasi-optimal schedules and values of objective are obtained by the algorithm, as shown in Table 2. This result provides a reasonable scheme for the project manager to make optimal scheduling in the project time-cost trade-off problem.

Table 2. Optimal scheduling scheme.

Optimal activity list	(1, 2, 18, 19, 3, 4, 7, 5, 15, 6, 10, 23, 16, 20, 12, 21, 22, 8, 9, 13, 14, 17, 11, 24)
Duration	138
Total cost	1139260

4. Conclusion

In the real-life project, due to the influence of uncertain environment, managers should consider the trade-off between the completion time and cost. By describing the increased quantities of resource as uncertain variables, an uncertain resource constrained project scheduling problem is discussed in this paper. Then an uncertain optimal model was built with objective of minimizing the completion time and the cost with resource constrained based on uncertainty theory. To solve this model, we transformed the uncertain model into equivalent form and proved it. Finally, we used genetic algorithm to search quasi-optimal solution of the model and gave a numerical example to illustrate the validity of the model. In future research, We can also focus on more types of project scheduling problems based on uncertainty theory.

Acknowledgements

This work was supported by the National Natural Science Foundation of China (No. 61873084), the Foundation of Hebei Education Department (No. ZD2017016), and the Innovation Fund for Graduate Students of Hebei Province (CXZZSS2019073).

References

- [1] Mori M, Tseng C C. A genetic algorithm for multi-mode resource constrained project scheduling problem [J]. *European Journal of Operational Research*, 1997, 100 (1): 134-141.
- [2] Peng W L, Wang C G. A multi-mode resource -constrained discrete time-cost tradeoff problem and its genetic algorithm based solution [J]. *International Journal of Project Management*, 2009, 27 (6): 600-609.
- [3] Ghoddousi P, Eshtehardian E, Jooybanpour S, Jaranmardi A. Multi-mode resource-constrained discrete time-cost-resource optimization in project scheduling using non-dominated sorting genetic algorithm [J]. *Automation in Construction*, 2013, 30: 216-227.
- [4] Browning T R, Yassine A A. Resource-constrained multi-project scheduling: Priority rule performance revisited [J]. *International Journal of Production Economics*, 2010, 126 (2): 212-228.
- [5] Lova A, Tormos P. Analysis of scheduling schemes and heuristic rules performance in resource-constrained multiproject scheduling [J]. *Annals of Operations Research*, 2001, 102 (1-4): 263-286.
- [6] Abbasi B, Shadrokh S, Arkat J. Bi-objective resource-constrained project scheduling with robustness and makespan criteria [J]. *Applied Mathematics & Computation*, 2006, 180 (1): 146-152.
- [7] Al-Fawzan M A, Haouari M. A bi-objective model for robust resource-constrained project scheduling [J]. *International Journal of Production Economics*, 2005, 96 (2): 175-187.
- [8] Ying Y. Research on the uncertain resource-constrained project scheduling problem [D]. Zhejiang University, 2010. In Chinese.
- [9] Xie F, Xu Z, Yu J. Bi-objective optimization for the project scheduling problem with variable resource availability [J]. *Systems Engineering-Theory & Practice*, 2016, 36 (3): 674-683. In Chinese.
- [10] Lambrechts O, Demeulemeester E, Herroelen W. Proactive and reactive strategies for resource-constrained project scheduling with uncertain resource availabilities [J]. *Journal of Scheduling*, 2008, 11 (2): 121-136.
- [11] Chen Z. Research on project scheduling problem with uncertain resource constraint [D]. Harbin University of Science and Technology, 2008. In Chinese.
- [12] Liu B. *Uncertainty Theory* [M], 2nd edn, Springer-Verlag, Berlin, 2007.
- [13] Liu B. *Theory and Practice of Uncertain Programming* [M]. Springer Berlin Heidelberg, 2009.
- [14] Ji X, Yao K. Uncertain project scheduling problem with resource constraints [J]. *Journal of Intelligent Manufacturing*, 2017, 28 (3): 575-580.
- [15] Ma W, Che Y, Huang H, Ke H. Resource-constrained project scheduling problem with uncertain durations and renewable resources [J]. *International Journal of Machine Learning and Cybernetics*, 2016, 7 (4): 613-621.
- [16] Liu B. *Uncertainty Theory: A Branch of Mathematics for Modeling Human Uncertainty* [M]. Springer-Verlag, Berlin, 2010.